

*The Measurement of Dielectric Constants by the Oscillations of
Ellipsoids and Cylinders in a Field of Force.*

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1. This paper deals with the measurement of dielectric constants by a method which, though simple, does not appear to have been previously employed in the same form. Ellipsoids and cylinders accurately cut from crystals and other insulating material are suspended between flat plates connected to an alternating transformer supplying a sinuous electrification of 1500 to 2000 volts amplitude, and the period of small swings of the suspended rod or ellipsoid, with and without the field, is observed. From these measurements the dielectric constant can be found.

The method is capable of a high order of accuracy, and the results obtained agree closely with the best of previous determinations, especially

those made by Hopkinson for different varieties of glass. It also lends itself to work with liquids, and some examination can be made of the polarisation of dielectrics in a unidirectional field.

The several methods which have been used in the measurement of the dielectric constants of insulators are fully described by Gray* and by Winkelmann.† A complete bibliography of the subject is given by the latter and by Landolt and Börnstein.‡ In most of those methods recorded, observations have been made of the change in the electrified state of charged plates when the dielectric substance is introduced between them. Boltzmann determined the constants of solids by measuring the translational force on a small sphere suspended in a field of force of known intensity; and Quincke those of liquids by the tension and pressure caused in them by electrification. In the foregoing the dielectric was in every case at rest; and with the exception of Boltzmann's experiments, the effects observed have been external to the cell containing the dielectric. The method now to be described differs from these in that the dielectric is ellipsoidal or cylindrical in form, and makes small oscillations about a position of symmetry in line with the field.

The apparatus is really a condenser, with a portion of its dielectric free to move. The electrostatic field was maintained in sine wave alternation at a frequency of about 80 a second; the oscillations of the rods never exceeded one a second. By the use of an alternating field, the influence of "elastic afterworking" and of conduction currents are avoided, and the conditions are more nearly those for which the constants are usually required.

After some preliminary experiments, a short reference to the method was made in an inaugural address.§ It was found later that Beaulard was at the time of the early experiments investigating the dielectric constant of water by enclosing it in a hollow glass ellipsoid suspended in a field maintained by the usual spark device in a state of oscillation at two million alternations a second. His value is very interesting, and is probably of the right order, though the experiment did not aim at a high degree of accuracy.||

The differences between the recorded values of the constant K are not on the whole to be explained by differences of composition. In view of the importance of its accurate determination under alternating fields of low frequency or infinite wave-length, the following measurements were

* 'Absolute Measurements in Electricity and Magnetism,' vol. 1, chap. VII, § 2.

† 'Handbuch der Physik' (2te Auflage), vol. 4, (1), pp. 92—150.

‡ Landolt and Börnstein, 'Phys. Tabellen,' Tab. 240.

§ 'Journal Inst. Elec. Engrs.,' 1905, Part 176, vol. 36, p. 186.

|| 'Science Abstracts,' No. 2097, November, 1905; 'Revue Electrique,' August 30, 1905.

made, in which the solid ellipsoids and cylinders used were worked into shape with all possible care. Great pains were taken to obtain a steady voltage, and to constantly compare it with a standard unidirectional voltage.

The points in the paper which perhaps call for special note are the small mass of substance used, in the solids not exceeding a third of a gramme and in liquids a tenth of a gramme; the close agreement between theory and experiment in § 5; and the practical method adopted of dealing with cylindrical specimens.

2. The restoring couple upon a dielectric ellipsoid with its axis inclined to an electrostatic field of force is a quadratic function of its dielectric constant.* When the ellipsoid is freely suspended by an elastic fibre, the couple may be measured by the rotation of a torsion head required to bring the body to any fixed angle with the direction of the field, or by the period of small oscillations about the position of rest. The motion in the latter case is that of a "quadrantal pendulum," the restoring moment of which reaches a maximum at an angular displacement of 45 degrees. The general theory of this motion has been given by Thomson and Tait,† and a solution for large amplitudes by Greenhill,‡ but in the above case there is, in addition to the polarisation couple, that of the elastic control.

From measurements of the couple, the dielectric constant can be found in terms of the strength of the field, the dimensions of the body, the constant of the surrounding medium, and the time of oscillation or angle of twist of the suspension fibre.

When the medium is air or vacuum, the method is absolute in the sense that the constant is determined from one set of measurements.

The equation of motion of a quadrantal pendulum under torsional control and with negligible damping is

$$I\ddot{\theta} + a\dot{\theta} + b \sin \theta \cos \theta = 0, \quad (1)$$

where I is the moment of inertia about the axis of suspension, a the restoring moment of the suspension per unit twist, and $b \sin \theta \cos \theta$ the polarisation couple, θ being the angle made by the longer axis with the undisturbed field.

In the case of an isotropic dielectric whose constant is K in a medium of constant K_0 ,

$$b = F^2 V \frac{(K - K_0)^2 (L - N)}{\{4\pi + (K - K_0)L\} \{4\pi + (K - K_0)N\}}, \quad (2)$$

where V is the volume of the body, F the intensity of the field in electro-

* Maxwell, 'Electricity and Magnetism,' vol. 2, §§ 437 *et seq.*

† 'Natural Philosophy,' vol. 1, § 322.

‡ 'Applications of the Elliptic Functions,' A. G. Greenhill, p. 103.

static units, L and N coefficients depending only upon the shape of the body.

In the case of a prolate ellipsoid of eccentricity e ,*

$$L = 2\pi \left(\frac{1}{e^2} - \frac{1-e^2}{e^3} \log \frac{1+e}{1-e} \right), \text{ the transverse coefficient,}$$

$$N = 4\pi \left(\frac{1}{e^2} - 1 \right) \left(\frac{1}{2e} \log \frac{1+e}{1-e} - 1 \right), \text{ the longitudinal coefficient.} \quad (3)$$

When the oscillations are so small that the square of the amplitude may be neglected,

$$I\ddot{\theta} + (a+b)\dot{\theta} = 0. \quad (4)$$

The period is therefore

$$T = 2\pi \sqrt{\left(\frac{I}{a+b} \right)}, \quad (5)$$

and

$$(a+b) = 4\pi^2 n^2 I.$$

The free period $T_0 = 2\pi\sqrt{I/a}$, so that

$$a = 4\pi^2 n_0^2 I \quad \text{and} \quad b = 4\pi^2 (n^2 - n_0^2) I. \quad (6)$$

Hence from equation (2)

$$\frac{(K-K_0)^2 F^2 V (L-N)}{\{4\pi + L(K-K_0)\} \{4\pi + N(K-K_0)\}} = 4\pi^2 (n^2 - n_0^2) I, \quad (7)$$

which gives

$$(K-K_0)^2 \left\{ \frac{F^2}{n^2 - n_0^2} \frac{V}{I} \frac{(L-N)}{4\pi^2} - LN \right\} - (K-K_0) 4\pi (L+N) - 16\pi^2 = 0, \quad (8)$$

the equation by which K is determined when K_0 is known. In this $F^2/(n^2 - n_0^2)$ is the term derived from the movement of the ellipsoid, the other terms are found from its dimensions and mass.

When the amplitude is so large that the period is sensibly increased, this must be reduced to the case of infinitely small swings.

$$\text{Writing} \quad \ddot{\theta} + \frac{a}{I}\dot{\theta} + \frac{b}{2I} \sin 2\theta = 0, \quad (9)$$

and with an amplitude α , we have

$$t = \sqrt{I} \int \frac{d\theta}{\sqrt{\{(a\alpha^2 - \frac{1}{2}b \cos 2\alpha) - a\theta^2 + \frac{1}{2}b \cos 2\theta\}}}.$$

Expanding $\cos 2\theta$, and rejecting terms from θ^6 upwards, we have for the period

$$T = 4 \sqrt{\left(\frac{3I}{b} \right)} \int_0^\alpha \frac{d\theta}{\sqrt{(\gamma - \beta\theta^2 + \theta^4)}}, \quad (10)$$

where

$$\beta = 3 \left(\frac{a}{b} + 1 \right), \quad \gamma = 3 \left(\frac{a}{b} \alpha^2 + \sin^2 \alpha \right). \quad (11)$$

* *Vide* Maxwell, *loc. cit.*

Writing $\frac{1}{p^2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta^2}{4} - \gamma\right)}, \quad \frac{1}{q^2} = \frac{\beta}{2} - \sqrt{\left(\frac{\beta^2}{4} - \gamma\right)}.$

$$T = 4p \sqrt{\left(\frac{3I}{b}\right)} \int_0^{\pi/2} \frac{d\phi}{\sqrt{(1-k^2 \sin^2 \phi)}}, \quad (12)$$

where $\sin \phi = p\theta$, and $k = p/q$.

For indefinitely small amplitudes γ , and therefore k , are zero, and in this case

$$p = \frac{1}{\sqrt{\beta}}, \quad T = 4 \sqrt{\left(\frac{3I}{b\beta}\right)} \cdot \frac{\pi}{2}. \quad (13)$$

For finite amplitudes

$$T_1 = \frac{4}{\sqrt{\left\{\frac{1}{2}\beta + \sqrt{\left(\frac{1}{4}\beta^2 - \gamma\right)}\right\}}} \cdot \sqrt{\left(\frac{3I}{b}\right)} \cdot F(\eta, \tfrac{1}{2}\pi), \quad (14)$$

in which $\sin \eta = k$, and the square of the frequency of oscillation reduced to indefinitely small amplitudes is therefore

$$n_1^2 = n^2 \frac{1}{\frac{1}{2} + \sqrt{\left(\frac{1}{4} - \gamma\beta^{-2}\right)}} \left\{ 1 + \frac{\frac{1}{2} - \sqrt{\left(\frac{1}{4} - \gamma\beta^{-2}\right)}}{\frac{1}{2} + \sqrt{\left(\frac{1}{4} - \gamma\beta^{-2}\right)}} \right\}^2, \quad (15)$$

which for most practical purposes may be written

$$n_1^2 = n^2 (1 + \tfrac{3}{2}\gamma\beta^{-2}). \quad (16)$$

The value of β upon which k depends, itself requires a knowledge of b . If it is impossible to work with small amplitudes this is a real difficulty, but if, as in the following experiments, the swing rarely exceeded 5 degrees, the reduction to zero amplitude may be omitted in a first approximation, and b calculated from equation (6). This value may then be used to find β , and the reduction made as above. If a still closer approximation is required the process may be repeated.

The logarithmic decrement of the motion was in most cases 0.06. The correction of the square of the frequency for this increases it in the ratio $(1 + \lambda^2/\pi^2)$, that is 1.00036.

The experiment then consists in observing n , n_0 , and F , correcting n for amplitude and damping, and inserting the value of $F^2/(n^2 - n_0^2)$ in equation (8), to calculate the value of K .

Arrangement of Experiment.

3. The ellipsoids were suspended by a quartz fibre, about 0.001 cm. diameter and 30 cm. long, between square brass plates b, b held apart by glass distance pieces g, g at their corners (see fig. 1) and enclosed in a glass vessel having a thick glass top carrying a suspension tube, torsion head, and circular scale. To reduce damping and to accelerate the drying of the specimens the vessel was in most of the determinations exhausted to

a vacuum of 700 mm. and a dish of phosphoric anhydride placed at the bottom. The size of the plates and their distance apart were chosen after trial with the stream line method* to determine the best proportions of the field under the distorting influence of the ellipsoids. With dielectric ellipsoids 2 cm. long a convenient distance is 6 cm. apart, with plates 10 cm. square. At this distance the feeble induced charges on the suspended body were, as shown by the results obtained for flint glass and quartz, insufficient

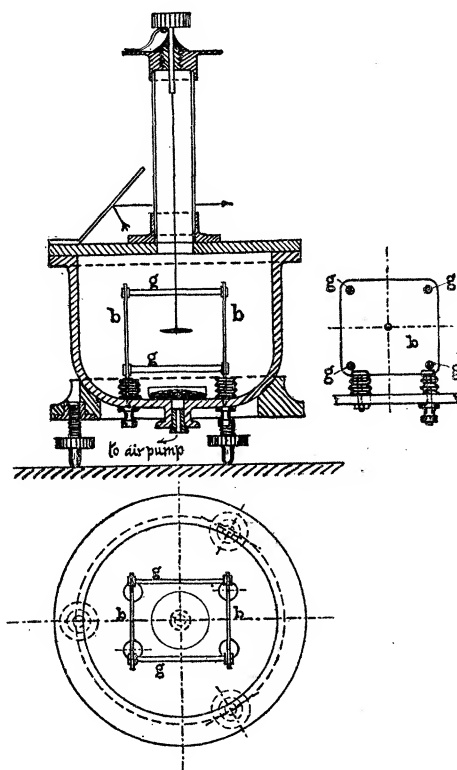


FIG. 1.—Testing Cell.

to change the distribution of electricity on the plates by any appreciable amount. At 4 cm. apart the period is increased by a few hundredths of a second. When, however, a metallic ellipsoid of the same size is placed in the field it is necessary to space the plates 10 cm. apart to reduce its reaction on them to less than a measurable amount. The motion of the specimen was observed by reflexion from a mirror into a telescope having an eye-piece with five cross wires radiating from the centre at angles of

* Hele Shaw and Hay, 'Phil. Trans.,' vol. 195, ser. A, pp. 307—327.

10 degrees apart. They were set with regard to the field by focussing the central one upon the ellipsoid brought to rest in a strong field of force.

The potential difference between the plates could be raised to 10,000 volts obtained from a small oil-cooled transformer. In order to get very steady voltages the steam end of a 24-kilowatt Parsons turbogenerator was disconnected from the generator so that the latter could be run independently as a motor. Slip rings were fitted over part of the commutator, and the machine driven from large storage cells as an inverted converter (fig. 2).

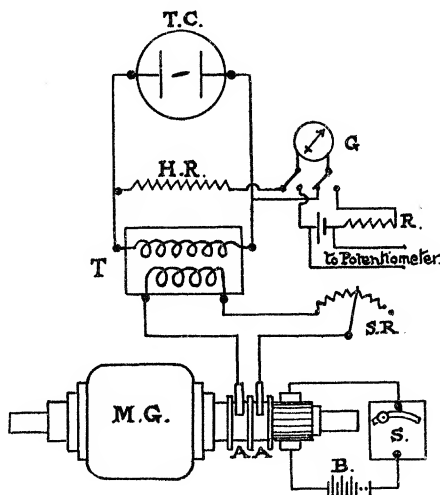


FIG. 2.—Arrangement of Apparatus.

- | | |
|---------------------------------------|---|
| B., storage battery. | T., step-up transformer. |
| S., motor starter. | H.R., 99,000 ω resistance. |
| M.G., motor generator. | G., hot wire galvanometer. |
| A.A., alternating current slip rings. | R., standard resistance, 100 ω . |
| S.R., switch of current regulator. | T.C., testing chamber. |

All slight flickering of the alternating voltage which had previously given trouble was in this way avoided, the maximum variation of pressure during several minutes—the time of observation of the period of the ellipsoids—being, as a rule, one part in 800.

Difficulty was for some time experienced in obtaining readings of the voltage consistent to 0.1 per cent. Several forms of voltmeter were tried without success. The means eventually adopted was to connect a high resistance across the terminals in series with a Duddell twisted strip ammeter which was enclosed in a metal case surrounded by flannel. It was found necessary, on account of the influence of slight changes of temperature upon the readings, to calibrate with direct current before and after each set

of observations with alternating current. When this was done very consistent readings were obtained.

Working by the method of oscillations the period was increased by air resistance to the extent of about 4 parts in 1000. The damping in every case was so small that a correction could be made by observation of the logarithmic decrement as indicated in § 2. Taking this into account no measurable effect was found from the change of the ionised state of the surrounding air when the pressure was reduced to 755 mm. of mercury, below which it was found not advisable to go on account of the occurrence of luminous discharge between the plates.

In working, a torsional swing was given to the ellipsoid, from the period of which and the known moment of inertia the elastic constants of the suspension were found. The voltage was then applied and the new period found. The ratio of the square of the intensity of the field to the difference of the squares of the frequency of oscillation with and without the field was the object of the experiment.

The glass and quartz ellipsoids and cylinders were suspended by forming a small drop of fused canada balsam on the quartz fibre, slightly warming the surface of the solid and quickly melting the drop into contact with it by the approach of a hot rod. If the ellipsoid were then out of balance, as tested on a levelled mirror, the attachment was broken and the process repeated, usually a dozen or more times before a perfect suspension was obtained. The other substances were hung in a cocoon silk saddle.

4. The substances selected for trial of the method were optical flint glass of densities 4·67, 4·1, and 3·3, and quartz cut parallel and perpendicular to the optical axis. The former were chosen because their constants have been very fully determined, especially by Hopkinson,* and the latter on account of the calculations of their dielectric constants which have been made, from very accurate optical observations, upon the electromagnetic theory of dispersion. Ellipsoids of revolution of each of these substances were made by Messrs. Hilger, and, tested by optical enlargement, were found to agree so closely with the true elliptic section that no difference could be measured between a photographic enlargement, by projection, of the ellipsoids, and a calculated ellipse, having the same major axis, 10 inches.

The ellipsoids were carefully measured, their volumes and densities determined, and their moments of inertia compared with a standard cylinder of heavy glass also made by Hilger, which together with a shorter cylinder was used in the determination of the influence of the ends of cylinders upon the polarisation torque. The measurements are given in Table I. It can be

* 'Original Papers,' vol. 2.

shown* that, on account of the low inductive constants of dielectrics, the influence of slight deviations from the true form does not rise in magnitude in the result.

Table I.—Measurements of Glass and Quartz Ellipsoids.

Substance.	Mass.	Volume.	Density.	Length.	Diameter.	L.	N.	I.
	gramme.	c.c.		cm.	cm.			gm.cm. ²
Quartz 	0·1420	0·0534	2·659	2·013	0·2235	6·1336	0·3013	0·02913
„ ⊥	0·1491	0·0560	2·663	2·014	0·2311	6·1255	0·3132	0·0308
Flint glass	0·2527	0·0543	4·654	2·022	0·2243	6·1276	0·3055	0·0519
„	0·2363	0·0573	4·12	2·006	0·2352	6·117	0·3292	0·04747
„	0·2091	0·0633	3·34	2·015	0·2452	5·848	0·3432	0·04360

Quartz.

5. When a uniaxal crystalline ellipsoid is suspended in a field of force, the restoring couple upon it depends upon the inclinations of the optic axis both to the direction of the field and to the axis of suspension.

The general expression for the couple upon an æolotropic ellipsoid is, with the previous notation,†

$$F^2V \left[\frac{4\pi (K_1 - K_2) + (K_1 - K_0) (K_2 - K_0) (L - N)}{\{4\pi + (K_1 - K_0) L\} \{4\pi + (K_2 - K_0) N\}} \right] \sin \theta \cos \theta. \quad (17)$$

For an ellipsoid cut perpendicular to the optic axis this will be greatest when the optic axis is parallel to the suspension. In this case K_1 and K_2 are each equal to $K_{\perp}$, the coefficient of dielectric polarisation at right angles to the optic axis. When the ellipsoid is cut parallel to the axis, the coefficient of longitudinal polarisation is $K_{\parallel}$, the transverse coefficient $K_{\perp}$. We proceed therefore to find first the dielectric constant for the ellipsoid cut perpendicular to the optic axis.

Quartz, perpendicular to the optic axis—

Ellipsoid.—Length 2·0145 cm., diameter 0·2311 cm., mass 0·149 gramme, volume 0·0560 c.c., density 2·66. $L = 6·1255$, $N = 0·3132$, $I = 0·0308$ gm.-cm.²

A series of observations was made with the optic axis inclined to and parallel to the axis of suspension, giving the following values, each of which is the mean of from 20 to 30 observations. θ is the angle between the two above axes in degrees.

* *Vide* Chrystal, 'Encyc. Britannica,' ed. 9, Art. "Magnetism," p. 245.

† 'Encyc. Britannica,' *loc. cit.*, eq. (82).

θ .	$\frac{F^2}{n^2 - n_0^2}$
0	139.25
-45	143.20
+45	141.90
90	141.10

A curve of these is drawn in fig. 3, the dip in which points to a marked change in the apparent electrical behaviour of the crystal when the polarisation is everywhere exactly at right angles to the optic axis, *i.e.*, the condition under which the ordinary ray is transmitted. The couple on the ellipsoid is a maximum at this point, but the approach to it is so steep that a very slight angular displacement from the axis of suspension causes a marked difference in the period. A calculation of the variation of the couple on the ellipsoid with the angle θ shows a dip of this nature but not quite so steep as in fig. 3.

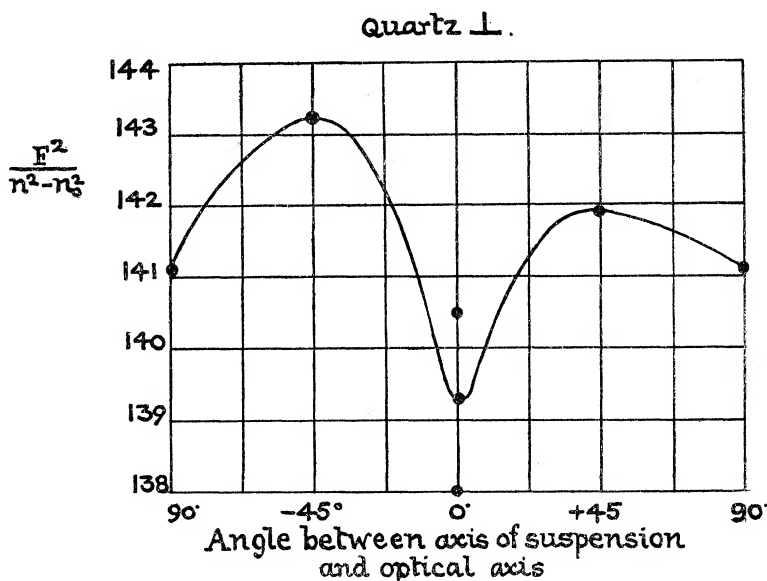


FIG 3.—Change of horizontal couple with vertical inclination of optical axis.

The value 139.25 is the mean of two series taken with different suspensions, the greatest care being taken to have the axes in line. The average value of the first series was 138 and of the other 140.5. It was impossible to obtain perfect alignment, a difference of 0.01 mm. on the circumference corresponding to an angular displacement of 4 degrees.

The determining equation (8) for this ellipsoid is

$$(139.25 \times 0.2676 - 1.9185)(K - K_0)^2 - 80.91(K - K_0) - 157.9 = 0,$$

from which

$$K - K_0 = 3.5490.$$

Taking K_0 for a vacuum of 700 mm. = 0.9995,

$$\mathbf{K1 = 4.5485.}$$

The following observations were made on the ellipsoid of quartz cut parallel to the optic axis over an interval of six months with the same suspension in each case. Each row is the mean of ten readings, but the pressure was not at this time very steady:—

Table II.

D.	<i>i</i> .	P.D.	F ² .	T.	n^2 .	$\frac{F^2}{n^2 - n_0^2}$.
38.31	0.0201	1990	1.24	9.495	0.01109	135.06
30.65	0.01787	1769	0.9801	10.425	0.009201	134.44
38.07	0.01983	1963	1.2069	9.57	0.01092	133.84
33.87	0.01872	1853	1.0750	10.03	0.00994	133.88
40.20	0.02045	2023	1.2816	9.35	0.01144	134.48
31.30	0.01801	1783	0.9955	10.36	0.00931	134.52
41.76	0.02087	2066	1.3372	9.187	0.01185	134.52
37.93	0.01988	1969	1.2129	9.553	0.01096	134.02
Mean.....						134.34

Here D is the observed hot-wire galvanometer deflection; *i* the corresponding current; P.D. the potential difference across the plates; F the electrostatic intensity of the field calculated from the voltage, assuming "*v*" = 3×10^{10} ; *n* the frequency of swing.

In order to reduce n^2 to the frequency for zero amplitude, the reading nearest to the mean is taken, that is the second row in the table; we have then from equation (6)—

$$a = 4\pi^2 \times 0.02912 \times 0.00191 = 0.00219,$$

$$b = 4\pi^2 \times 0.02912 (0.009201 - 0.001910) = 0.0083605,$$

and from (11),

$$\beta = 3 \left(\frac{0.00219}{0.0083605} + 1 \right) = 3.7857,$$

$$\gamma = 3 (0.2619 \times 0.004873 + 0.0048658) = 0.01846.$$

Thus

$$\gamma/\beta^2 = 0.018426/14.3314 = 0.001285,$$

and by (16),

$$n^2 = 0.009201 \{0.5 + \sqrt{(0.25 - 0.001285)}\}^{-1} \left(1 + \frac{1}{4} \frac{0.5 - \sqrt{(0.25 - 0.001285)}}{0.5 + \sqrt{(0.25 - 0.001285)}} \right)^2,$$

that is $n^2 = 0.009215$. This is increased to allow for damping in the ratio 1.00036, giving finally $n^2 = 0.009218$. The corresponding value of $F^2/(n^2 - n_0^2)$ is 134.11. The influence of the correction has been to reduce the above quantity in the ratio 134.11/134.44, that is 0.9974. Applying, now, this reduction factor to the mean value 134.34, we obtain finally

$$F^2/(n^2 - n_0^2) = 134.0.$$

Measurements of the ellipsoid gave $I = 0.02913$, $V = 0.0534$ c.c., $L = 6.1336$, $N = 0.3013$. Thus, from equation (8),

$$(134.0 \times 0.2716 - 1.848)(K - K_0)^2 - 80.86(K - K_0) - 157.9 = 0,$$

from which $(K_1 - K_0) = 3.6073$, giving finally, with $K_0 = 0.9995$ and $K_{\perp} = 4.5485$,

$$K_{\parallel} = 4.600.$$

It is interesting to compare these results with those obtained from optical data by the Ketteler and Sellmeier dispersion formula—

$$K_{\infty} = \mu_{\lambda}^2 - \frac{M_1}{\lambda - \lambda_1^2} + \frac{M_2}{\lambda_2^2 - \lambda^2} + \frac{M_3}{\lambda_3^2 - \lambda^2},$$

where K_{∞} is the dielectric constant for exceedingly long waves; μ the index of refraction at wave-length λ ; M_1, M_2, M_3 , constants derived at the wave-lengths $\lambda_1, \lambda_2, \lambda_3$, at which selective absorption occurs. These constants have been found for quartz by Rubens and Nichols,* using a bolometer as a detector of waves coming from an Auer burner, reflected several times from the surfaces of quartz blocks, and separated by a grating. From their results the following values were obtained—

$$\begin{aligned} \lambda_1 &= 0.1031, & \lambda_2 &= 8.85, & \lambda_3 &= 20.75 \\ M_1 &= 0.01065, & M_2 &= 44.224, & M_3 &= 713.55, \end{aligned}$$

the wave-lengths being in 10^{-4} cm.

If μ is taken for the D line, and for the ordinary ray the formula gives $K(\text{ord.}) = 4.578$.

The above values of the constants M_1, M_2, M_3 are usually quoted as being appropriate to the ordinary ray. The constants for a similar dispersion formula for the extraordinary ray have not been calculated, but if μ is taken for the D line and for the extraordinary ray we get as an estimate $K(\text{ext.}) = 4.6065$.

The above formula has recently been slightly modified by Maclaurin,† who has worked out the constants for rocksalt and sylvia but not for quartz. It

* 'Wied. Ann.,' vol. 60, p. 418, 1897.

† 'Roy. Soc. Proc.,' A, vol. 81, p. 367, 1908.

is probable that the value 4.6065 would be reduced in conformity with his treatment by as much possibly as 0.2 per cent.

In comparing results obtained from optical data with those derived from direct electrical experiment, the electric polarisation must be in the same direction in both cases. Since the electrical polarisation in a light wave is transverse, values of K calculated by the above formula, using the index of refraction for the ordinary ray, obtained by the transmission of light along the optic axis, correspond to those found directly by exposing the quartz to an electric field at right angles to the axis. Thus K (ord.) as calculated should agree with $K_{\perp}$, and K (ext.) with $K_{\parallel}$. We have

Observed.	Calculated.
$K_{\perp}$ 4.548	K ord. 4.578
$K_{\parallel}$ 4.600	K ext. 4.606

It may be remarked that the mean of the observed values of $K_{\perp}$ and $K_{\parallel}$, which is 4.572, agrees better with the value of the corresponding constant in the dispersion formula than $K_{\perp}$.

There are two further points bearing upon the agreement of the observed and calculated values which must be considered. The expression "at right angles to the axis" does not specify the precise direction within the crystal in which the cutting was made. The value $K_{\perp}$ is therefore subject to a possible variation of a small fraction of 1 per cent. from the mean. Again, the composition of quartz is known to vary, but fine clear crystals are pure silica. Both Nichols and Rubens, and also Hilger, remark that the crystals used by them were very fine, and the agreement indicates that the molecular state was very nearly the same in both cases. There is no recorded investigation of the influence of change of composition on the dielectric constant of quartz, but judging by the small difference between the constants $K_{\parallel}$ and $K_{\perp}$, it would be small.

At the end of the paper, values of K obtained by Felsing, Ferry, Curie, and Romich and Nowak are given, the mean of which for $K_{\parallel}$ is 4.66 and for $K_{\perp}$ 4.54.

Fused Quartz.

The density of fused quartz is about 0.86 that of crystalline. In order to see how the dielectric constant depends upon density as distinct from structure, an ellipsoid was made of quartz, carefully fused and perfectly clear. It

had the following dimensions: diameter, 0.2291 cm.; length, 2.003 cm.; mass, 0.1265 gramme; volume, 0.055 c.c.; $L = 6.125$, $N = 0.311$, density, 2.30.

The mean value for $F^2/(n^2 - n_0^2)$ was 162.25, from which

$$(162.25 \times 0.316 - 1.904)(K - K_0)^2 - 80.82(K - K_0) - 157.9 = 0,$$

giving

$$K - K_0 = 2.78 \quad \text{and} \quad \mathbf{K} = 3.78.$$

The ratio of this to the mean of K_{II} and $K_{\perp}$ is 0.826. The change in the dielectric constant is from this about 4 per cent. less than the corresponding change in density: in keeping with the fact that the influence of the crystal-line structure in quartz upon the dielectric constant is comparatively small.

Flint Glass.

6. The dielectric constant of flint glass has been very fully investigated by Hopkinson,* using a guard-ring condenser.

It is common in modern optical work to use a heavy flint glass of density about 4.67. Its departure from Maxwell's law $K = \mu^2$ is great, but its large dielectric constant renders it a suitable material for a standard dielectric ellipsoid to be used in alternating fields of low frequency.

In order, however, to make a comparison over the whole range of Hopkinson's research three ellipsoids were made of densities 4.65, 4.1, and 3.3, the measurements of which are given in Table I.

For the first, diameter 0.2243 cm.; length, 2.022 cm.; mass, 0.2527; volume, 0.0543; $L = 6.1276$, $N = 0.3055$, $I = 0.0519$, the mean of many observations of $F^2/(n^2 - n_0^2)$ corrected for amplitude and damping gave the value 83.57.

We have then

$$(83.57 \times 0.1543 - 1.872)(K - K_0)^2 - 80.84(K - K_0) - 157.9 = 0,$$

from which

$$K - K_0 = 9.645.$$

In this experiment the vessel was not exhausted, so that

$$K_0 = 1, \quad \text{and} \quad \mathbf{K} = 10.645 \quad (\Delta = 4.65).$$

In order to fix the position of the curve between the densities 3.66 and 4.5, the ellipsoid of density 4.12 was made, the measurements of which gave—

$$\text{Volume} = 0.05736 \text{ c.c.}, \quad I = 0.04747, \quad L = 6.117, \quad N = 0.3292.$$

The mean value of $F^2/(n^2 - n_0^2)$, corrected as before, was 87.6 at a frequency of 87.5. Thus

$$(87.6 \times 0.1776 - 2.013)(K - K_0)^2 - 81.0(K - K_0) - 157.9 = 0,$$

giving

$$\mathbf{K} = 8.52 \quad (\Delta = 4.12).$$

* 'Phil. Trans.,' Part I, 1878, pp. 17—23; Part II, 1881, pp. 355—373.

The ellipsoid of light flint glass, of density 3.34, melting point 674° C., had

$$\text{Volume} = 0.0633 \text{ c.c.}, \quad I = 0.04360, \quad L = 5.848, \quad N = 0.3432.$$

The mean value of $F^2/(n^2 - n_0^2)$ was 94.65 at a frequency of 86.

Thus for this glass

$$(94.65 \times 0.2054 - 2.007)(K - K_0)^2 - 77.8(K - K_0) - 157.9 = 0,$$

from which

$$K = 6.98 \quad (\Delta = 3.30).$$

Comparing these values with Hopkinson's, there is seen to be very close agreement in the dense glasses, but not with the light glass, the difference in that case being 2.3 per cent. In working with light flint glass the composition and melting point must be known in order to make a fair comparison, because of the possibility of obtaining the same density by a variation of the proportions. The composition of the heavier glasses is more closely fixed by the proportion of lead required to give the greater density.

The symmetry of the curve of fig. 4 shows that the variation of density

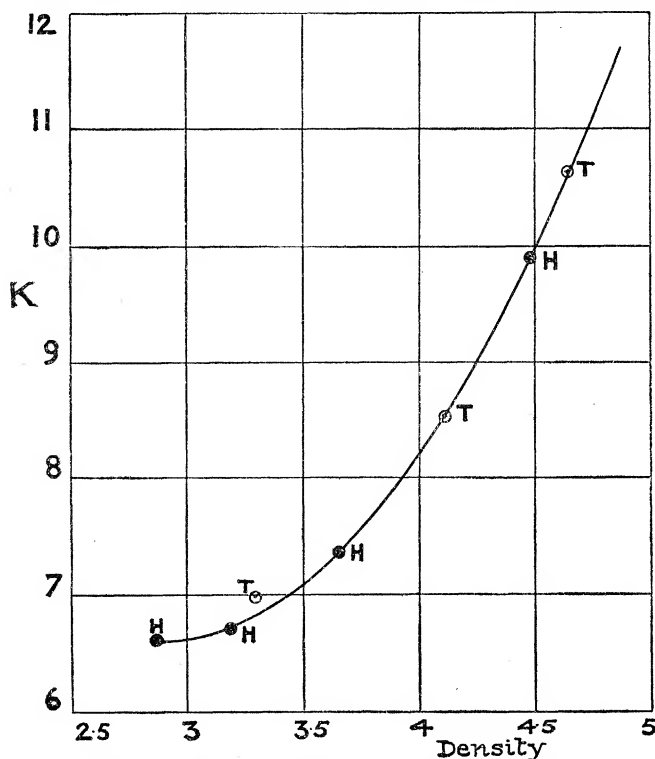


FIG. 4.—Variation of the dielectric constant of flint glass with density.

was obtained by a systematic change of the proportion of lead,* and while the curve is valuable in itself, especially near the origin, there must always

* Vide Hopkinson, *loc. cit.*

be a possible variation of several per cent. in the value of the dielectric constant of light flint glass from different makers at any given density.

7. The curve in fig. 4 is parabolic and agrees very well with

$$K - K_1 = b(d - d_1)^2, \quad (18)$$

where d is the density, K the dielectric constant, and $K_1 = 6.61$, $d_1 = 2.89$, $b = 1.2965$.

The density, 2.89, is that of nearly the lightest flint glass made. From this to the value 4.65 the change of the dielectric constant is proportional to the square of the change of density.

The relation between K and μ which is shown by fig. 5 is also a quadratic function expressed by

$$K - K_1 = c(\mu_F - \mu_1)^2, \quad (19)$$

in which

$$K_1 = 6.61, \quad \mu_1 = 1.549, \quad c = 95.6.$$

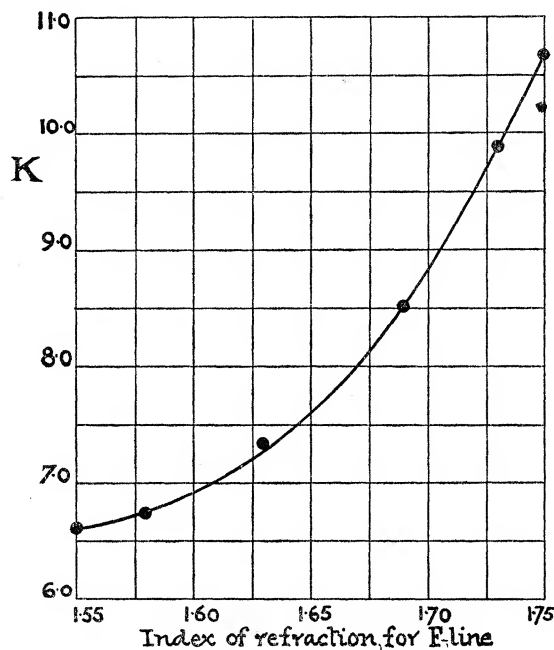


FIG. 5.—Dielectric constant and index of refraction. Flint glass.

The following table gives corresponding values of d , K , and μ , and the derived values of c :—

Table III.

$d.$	K.	$\mu_F.$	$c.$
2·8(7)	6·61	1·549	—
3·2	6·72	1·5838	95
3·66	7·3(7)	1·6347	95
4·12	8·52	(1·690)	96
4·50	9·89	1·735	95
4·65	10·645	(1·753)	96·9

From (18) and (19),

$$\mu_F - \mu_1 = g(d - d_1), \quad (20)$$

in which $\mu_1 = 1·549$, $d_1 = 2·89$, $g = 0·1155$.

The value 2·89 has been taken rather than 2·87, for the zero of the curve of density is nearer the former value, and if Hopkinson's experimental determinations of a in the empirical relation*

$$\mu - 1 = a + bf(\lambda)$$

are drawn with respect to the density, the density corresponding to $a = 0·549$ upon a straight mean line is 2·89.

The value 0·1155 of the coefficient g was calculated from Hopkinson's experiments, being the slope of the a , d line, but from 18, 19, and 20, $g = \sqrt{b/c}$.

If for c the last figure in the foregoing table is taken,

$$g = \sqrt{\frac{1·296}{96·9}} = 0·1156.$$

Moulded Ellipsoids.

8. Many substances for which the dielectric constant is required in practice cannot be conveniently worked when hard, but may be moulded. Paraffin wax, shellac, and gutta-percha are examples. To deal with these a mould was made by preparing two blocks of lead $1 \times 2 \times 4$ cm. Between these a steel ellipsoid, 3 cm. long, was placed, and the blocks crushed together in a powerful press. A semi-ellipsoidal cavity was in this way formed in each. Castings could not be made by pouring into the mould on account of the smallness of the cavity and the difficulty in removing air bubbles. The substances moulded were all of a kind which becomes plastic on heating. They were therefore warmed until, when placed between the plates and pressed, they took the form of the mould, any excess being squeezed out between the blocks. These were then separated and the excess

* 'Roy. Soc. Proc.,' No. 182, pp. 1—8, 1877.

removed, the blocks warmed, and the process repeated until no excess was found. On being removed they were then very fair ellipsoids, but in every case required trimming by scraping away of some excess along the joint. Substances which, like sulphur or resin, have a relatively high melting point, solidify quickly, and adhere to the mould even when smeared with oil or glycerine, could be more conveniently prepared by forming into cylinders, as described in §§ 15 and 16.

In every case great care had to be taken to thoroughly dry the specimen and the containing vessel. The latter was kept warm, and when it was necessary to admit air in order to remove the specimen after a test, this was done through tubes of phosphorus pentoxide. When moisture was suspected the specimens were always allowed to stand for several hours in the alternating field and *in vacuo*, in order by the internal heating caused by dielectric hysteresis to help to dry the surface.

Paraffin Wax.

9. Moulded ellipsoid of pure clear wax of density 0.914.—Melting point 55°·8 C., diameter 0.2895 cm., length 2.93 cm., volume 0.141 c.c., moment of inertia by comparison with standard glass cylinder 0.0607 gm.cm.², mass 0.1290 gramme, eccentricity 0.995, $L = 5.98$, $N = 0.2715$, temperature 17° C., vacuum 700, frequency 86. The free period was 52.5 seconds. Period 19.1 seconds in a field $F = 1.025$ units. $F^2/(n^2 - n_0^2) = 442.5$.

The determining equation is

$$\left(\frac{442.5 \times 0.141 \times 5.708}{39.4 \times 0.0607} - 1.623\right)(K - K_0)^2 - 12.566 \times 6.251(K - K_0) - 157.9 = 0,$$

and

$$K = 2.32_6.$$

For freshly-moulded wax the highest value found was 2.4. Increasing the frequency from 60 to 84 decreased K by 0.7 per cent.

Bees-wax.

10. Ellipsoid.—Diameter 0.3194 cm., length 3.005 cm., mass 0.1521 gramme, volume 0.160 c.c., density 0.951, $L = 5.9$, $N = 0.30$, temperature 18° C. Free period 73 seconds. Period 10.21 seconds in a field $F = 1.261$ units.

Here

$$F^2/(n^2 - n_0^2) = 169,$$

and

$$\left(\frac{169 \times 5.6 \times 1.41}{39.4} - 1.773\right)(K - K_0)^2 - 78(K - K_0) - 157.9 = 0,$$

so that

$$K = 4.75.$$

This wax, from a lemon-yellow cake, one of the Royal Agricultural Society's prize winners, was worked in warmed moulds, and dried *in vacuo* over phosphorus pentoxide for a week.

Shellac.

11. Ellipsoid.—Diameter 0.323 cm., length 3.1 cm., volume 0.169 c.c., mass 0.196 gramme, density 1.15, moment of inertia 0.0965, $L = 5.92$, $N = 0.3019$, temperature 17°C . Free period 40.4 seconds. Period 6.92 seconds in field $F = 1.16$ units. $F^2/(n^2 - n_0^2) = 66.22$.

$$\text{Thus} \quad 147.8(K - K_0)^2 - 78.05(K - K_0) - 157.9 = 0,$$

from which

$$\mathbf{K} = \mathbf{2.49}.$$

The shellac was dark orange flake before melting, and was carefully heated until only the most minute bubbles of air were found in it when solid. Like paraffin, shellac polarises quickly, and the higher values which have been obtained by previous experimenters may have been in some cases from too long contact.

Sealing Wax.

12. I. Ellipsoid.—Diameter 0.317 cm., length 3.05 cm., volume 0.1604 c.c., moment of inertia 0.1263 gm.cm.², mass 0.274 gramme, $L = 5.915$, $N = 0.299$. Free period 38.0 seconds. Period 11.73 seconds in $F = 1.155$ units. $F^2/(n^2 - n_0^2) = 203.1$.

The determining equation is

$$\left(\frac{203.1 \times 0.1604 \times 5.615}{39.4 \times 0.1263} - 1.769 \right) (K - K_0)^2 - 12.566 \times 6.214 (K - K_0) - 157.9 = 0,$$

from which

$$\mathbf{K} = \mathbf{4.56}, \text{ at a density } 1.63.$$

In making this ellipsoid, a stick of the best sealing-wax was melted once, and from the plastic mass sufficient taken to form the ellipsoid between hot moulds. Another ellipsoid was formed from the same wax, but the heating was inadvertently continued for half an hour. The density of the new ellipsoid was 1.68, and, measured in the same way as above, the dielectric constant had risen to 5.2. The wax contained vermilion, button lac, resin, and "mineral," about half being compounds of mercury and lead and half vegetable resins.

Gutta-percha.

13. This ellipsoid was made from new thin clear gutta-percha tissue (Silvertown best quality), dried, melted, and moulded. In melting the tissue to a homogeneous mass, the heat was continued for about half an hour, and was such that bubbles formed in the plastic substance. There is reason to believe that this heating was excessive, but the case is given as an example. It is possible that something of the nature of carbonisation may have taken place.

The ellipsoid was dried for several months over phosphoric anhydride, and

then exposed to an alternating field of about 300 volts per centimetre for four hours in a vacuum of 700 mm. before taking a reading. These precautions were taken, on account of the high values of K obtained, to reduce the risk of the presence of moisture.

The measurements were—

Diameter 0.317 cm., length 2.952 cm., volume 0.147 c.c., mass 1.35 grammes, density 0.92, $L = 5.9$, $N = 0.30$. Free period 50.6 seconds. Period in a field $F = 1.046$, 7.49 seconds. $F^2/(n^2 - n_0^2) = 62.8$.

Hence $\left(\frac{62.8 \times 2.46 \times 5.6}{39.4} - 1.77\right)(K - K_0)^2 - 78(K - K_0) - 157.9 = 0$,
giving $K = 6.3$ (see § 16).

Chatterton Compound.

14. The sample was of ordinary commercial insulating compound. Its composition was unknown, though it apparently consisted largely of gutta-percha and pitch.

Ellipsoid.—Diameter 0.315 cm., length 2.952 cm., volume 0.160 c.c., moment of inertia 0.083 gm. cm.², mass 0.1821 gramme, temperature 18° C., density 1.13, $L = 5.91$, $N = 0.299$. Free period 30.85 seconds. Period 10.3 seconds in $F = 1.187$ units. $F^2/(n^2 - n_0^2) = 151.24$.

Thus $\left(\frac{151.24 \times 0.16 \times 5.61}{39.4 \times 0.083} - 1.769\right)(K - K_0)^2 - 78(K - K_0) - 157.9 = 0$,
so that $K = 3.98$.

Dielectric Cylinders.

15. Many dielectrics are difficult to shape into ellipsoids even approximately true in form, which can be cast or turned to very accurate cylinders. The electrical intensity within a cylinder in a field of force is, however, not uniform, and the reaction coefficients L and N have, strictly speaking, no definite meaning for cylinders. If not more than three-figure accuracy is required, as in commercial testing, it may be assumed that the polarisation couple upon cylinders can be expressed by $b \sin \theta \cos \theta$, where b is of the form given in equation (2). If the cylinder is long compared with its diameter, we may take the transverse coefficient $L = 2\pi$, the value which it has for infinitely long cylinders. The value of the longitudinal coefficient N , derived from equation (7), is

$$N = \frac{\frac{(K - K_0)}{4\pi^2} \frac{F^2}{(n^2 - n_0^2)} \frac{V}{I} L - \frac{16\pi^2}{(K - K_0)} - 4\pi L}{\frac{(K - K_0)}{4\pi^2} \frac{F^2}{(n^2 - n_0^2)} \frac{V}{I} + 4\pi + (K - K_0) L},$$

and is therefore not independent of the dielectric constant. By measure-

ment of the polarisation couple upon cylinders of known dielectric constant, a series of empirical values of N can be obtained, covering the range of K required in practice, for all convenient ratios m of length to diameter.

The following measurements were made for the purpose of finding N in this way, and figs. 6 and 7 are drawn from them.

Metallic Cylinders (K infinite).

- (a) Length 2.433 cm., diameter 0.1604 cm., $m = 15.07$, $\Delta = 8.72$, $L = 6.283$. Free period more than 5 minutes. Period in a field $F = 1.147$, 4.44 seconds; from which $F^2/(n^2 - n_0^2) = 25.92$ and $N = 0.149$.
- (b) Length 1.571 cm., diameter 0.1604 cm., $m = 9.8$, $\Delta = 8.72$. Free period negligibly great. Period in $F = 1.15$, 3.98 seconds. $F^2/(n^2 - n_0^2) = 21.0$ and $N = 0.228$.
- (c) Length 0.96 cm., diameter 0.1604 cm., $m = 5.98$, $\Delta = 8.72$. Free period over a minute. Period in $F = 1.15$, 3.51 seconds. $F^2/(n^2 - n_0^2) = 16.27$ and $N = 0.376$.

Dense Flint Glass.

- (a) Length 3.014 cm., diameter 0.220 cm., $m = 13.68$, $L = 6.283$, $K = 10.75$, $\Delta = 4.68$. Free period 90.6 seconds. Period in a field $F = 1.174$, 13.61 seconds. Thus $F^2/(n^2 - n_0^2) = 271.5$; from which $N = 0.258$.
- (b) Length 2.004 cm., diameter 0.2258 cm., $m = 8.88$, $K = 10.65$, $\Delta = 4.65$. Free period 17.51 seconds. Period in $F = 1.219$, 8.41 seconds. $F^2/(n^2 - n_0^2) = 136.34$ and $N = 0.422$.
- (c) Length 1.504 cm., diameter 0.2189 cm., $m = 6.88$, $\Delta = 4.68$, $K = 10.75$. Free period (on silk suspension) several minutes. Period in $F = 1.147$, 8.02 seconds. $F^2/(n^2 - n_0^2) = 84.7$ and $N = 0.532$.

Light Flint Glass.

- (a) Length 3.009 cm., diameter 0.2295 cm., $m = 13.1$, $\Delta = 3.23$, $K = 6.92$. Free period several minutes. Period in $F = 0.805$, 22.78 seconds. Thus $F^2/(n^2 - n_0^2) = 336$ and $N = 0.34$.
- (b) Length 1.87 cm., diameter 0.2295 cm., $m = 8.14$, $\Delta = 3.23$, $K = 6.92$. Free period 69 seconds. Period in $F = 0.853$, 14.26 seconds. $F^2/(n^2 - n_0^2) = 148$ and $N = 0.56$.
- (c) Length 1.03 cm., diameter 0.2295 cm., $m = 4.48$, $\Delta = 3.23$, $K = 6.92$. Free period 27.6 seconds. Period in $F = 0.883$, 8.1 seconds. $F^2/(n^2 - n_0^2) = 54.7$ and $N = 0.88$.

Paraffin Wax.

The value previously found for paraffin wax is that for annealed wax free from structure. It was found to be practically impossible to obtain cylinders free from initial strain. The method adopted was to melt the wax and draw it into a short piece of barometer tube slightly warmed to prevent the wax solidifying at the circumference and shearing. With a little practice

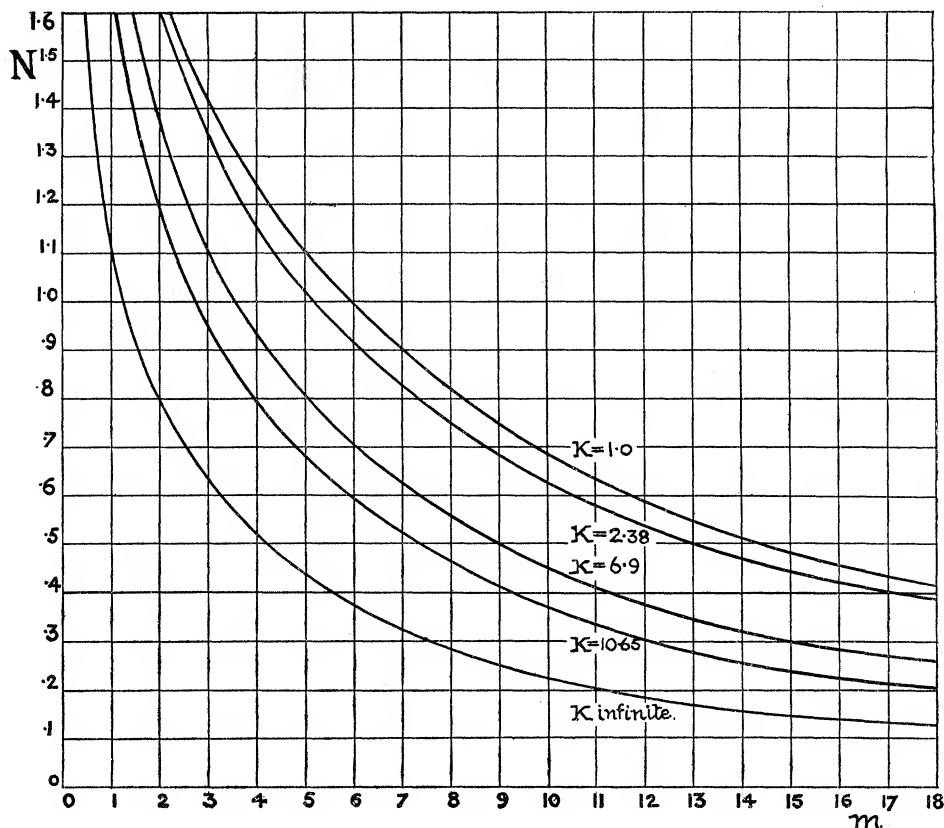


FIG. 6.—Variation of reaction coefficient N with m and K .
 m = length/diameter of cylinder.

cylinders were made, the diameter of which did not differ at any point by 0.001 inch from the mean.

(a) Length 3.56 cm., diameter 0.217 cm., $m = 16.36$. Free period 50.4 seconds. Period in field $F = 1.132$, 23.03 seconds. $F^2/(n^2 - n_0^2) = 871$, which with $K = 2.36$ gives $N = 0.41$.

(b) Length 2.64 cm., diameter 0.216 cm., $m = 12.28$. Free period 30.6 seconds. Period in field $F = 1.15$, 17 seconds. $F^2/(n^2 - n_0^2) = 550$ and with $K = 2.38$, $N = 0.55$.

- (c) Length 1.24 cm., diameter 0.217 cm., $m = 5.71$. Free period 10 seconds.
 Period in F = 1.232, 6.82 seconds. $F^2/(n^2 - n_0^2) = 135.5$, giving with
 $K = 2.38$, $N = 0.95$.

Fig. 6 is a record of these experimental values of N , but in finding the appropriate constant to use in any particular case, fig. 7 will be more useful. It is not necessary, however, to know any of these curves with great accuracy, for in working out the following tests the procedure was to assume any likely value for N , having regard to the dimensions of the specimen

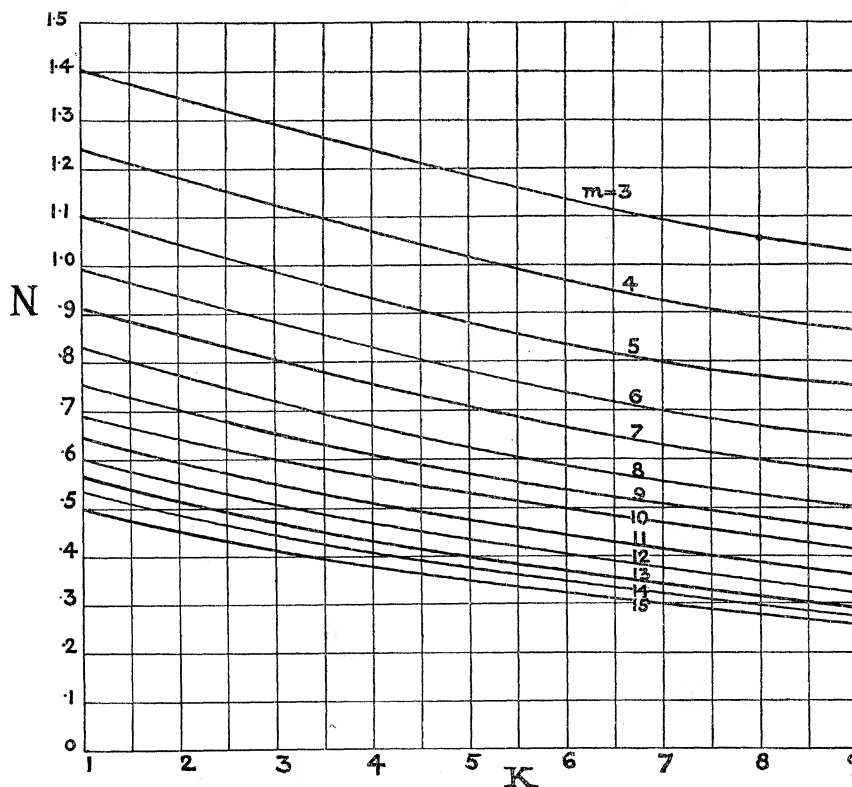


Fig. 7.—Change of N with K for different ratios of length to diameter of cylinder.

and the nature of the substance, and to calculate the corresponding dielectric constant, then from the curves to find N for the new value of K , and in this way by successive approximation to reach a point where the changes became less than experimental error. The use of the curves facilitates the finding of K by this method of trial and error. The value of N taken as determining that of K was usually found at the third approximation. When the point K , N for the given ratio m agreed with the curves of figs. 6 or 7 it was taken as final.

16. The cylinders were shaped by turning and moulding. That of *ebonite*, $\Delta = 1.22$, was turned from a sheet 1 cm. thick of the best Silvertown material, and gave a value 2.79. A very clear sample of *amber*, $\Delta = 1.06$, was turned by holding fine sand-paper pressed between the fingers upon the specimen, and gave 2.80. A cylinder turned from a piece of old *ivory*, $\Delta = 2.0$, gave $K = 6.9$. For *canada balsam*, $\Delta = 1.025$, a pale clear yellow sample melted and cast, cooling slowly, $K = 2.72$. For *resin*, $\Delta = 1.089$, prepared in the same way, a clear light brown sample, $K = 3.09$. Sublimed *sulphur*, $\Delta = 1.99$, heated to melting point and cast in a glass tube, cooling slowly with radial structure not reaching quite to the centre, gave $K = 4.03$. A *gutta-percha* cylinder, $\Delta = 0.815$, was prepared by rolling gently a small piece cut from a rod of Silvertown "super" quality warmed over an incandescent lamp, giving $K = 4.43$. A cylinder of pure soft black *indiarubber*, obtained from the makers and tested in fields of different intensities to eliminate electrostriction, gave finally $K = 3.08$.

Olive Oil.

17. The sample had a density 0.915 at 20°. It was dried by shaking with fused calcium chloride.

In order to eliminate as far as possible the influence of the necessary envelope, a tube was made by rolling a piece of cigarette paper 2 cm. square around an oiled steel cylinder having 0.238 cm. diameter. The oil caused the three layers used to adhere by capillarity, and the tube was dipped in the oil and wound up tightly on the cylinder. Made in this way a tube has sufficient stiffness to retain its shape perfectly when suspended filled with liquid.

The measurements were as follows:—

Length of tube 2.00 cm., diameter 0.238 cm., thickness 0.0023 cm., mass of tube 0.0055 gramme, mass of oil contained 0.0814 gramme, ratio of length to bore 8.3. Free period of paper tube alone 7 seconds. Period in field 6 seconds. Free period full of oil 31.6 seconds. Period in a field $F = 1.065$, 11.06 seconds. The moment of inertia of the cylinder alone $I_1 = 0.001926$ and of the oil alone 0.0272 gm.cm.².

The polarisation couple is therefore

$$39.4(0.0272 \times 0.00718 - 0.0000132) = 0.00717 \text{ dyne-cm. per radian.}$$

The volume of the oil was 0.089 c.c.

Assuming by trial a depolarising factor $N = 0.7$, we have

$$(K - K_0)^2 \left(\frac{1.135 \times 0.089 \times 5.58}{0.00717} - 4.39 \right) - 12.56 \times 6.28 (K - K_0) - 157.9 = 0,$$

from which

$$K = 3.16.$$

The average value of all the determinations quoted by Winkelmann is 3.106. Hopkinson found 3.16. The commonest adulterant of olive oil is cottonseed oil, which has a dielectric constant of 3.88.

Heavy "White" Paraffin Oil.

18. Used for electric condensers, having an exceedingly high resistivity. Density, 0.885 at 27°·5 C.

Paper tube 1.99 cm. long, 0.238 cm. diameter, mass of liquid used 0.0787 gramme, ratio of length to diameter 8.37. Free period of tube filled with oil 25.7 seconds. Period in a field $F = 1.098$, 12.15 seconds. Polarisation couple 0.004735 dyne-cm./radian, and the equation with $N = 0.71$.

$$121.84 (K - K_0)^2 - 87.9 (K - K_0) - 157.9 = 0,$$

giving

$$K = 2.55.$$

For this oil

$$\mu_D = 1.484, \quad \mu_D^2 = 2.2.$$

Water.

19. Proceeding as before, distilled water was enclosed in a paper tube 2.02 cm. long, 0.238 cm. diameter, or, allowing for the wetting of the paper, 0.24 cm.

Mass of water in tube 0.0884 gramme. Free period, full, 27.6 seconds. Period in a field $F = 1.075$, 1.76 seconds. Polarisation couple 0.364 dyne-cm./radian.

From this the cylinder had apparently an infinite dielectric constant, which means that at the frequency of 80 a second electrical conduction has time to come into full operation.

Collected Results.

20. For convenience of reference the results obtained in the paper are collected below, together with some previous determinations at frequencies not widely different from those of the paper. There is on the whole close agreement between the new values and those quoted. The differences between the best previous determinations are from 2 to 5 per cent. By the method of the paper an accuracy of one-tenth per cent. is obtained without difficulty except that of moisture or other conducting agency. The work has been extended to the slow polarisation of dielectrics in unidirectional fields.

I wish to thank my late students, Mr. O. J. Williams, B.Sc., and Mr. B. J. M. Lane, B.Sc., for much help, especially in the development of the method, and my friend, Dr. T. H. Havelock, for several valuable suggestions.

Table IV. ($\sim = 80$ to 85.)

Substance.	K.	Some previous determinations.
Quartz 	4·600	5·06, Fellingner; 4·46, Ferry; 4·55, Curie; 4·6, Romich and Nowak. Mean of these, 4·66.
Quartz ⊥.....	4·548 ₅	4·69, Fellingner; 4·38 Ferry; 4·49, Curie; 4·6 Romich and Nowak. Mean of these, 4·54.
Fused quartz	3·78	
Flint glass—		
$\Delta = 4·65$	10·64	10·65 } 8·52 } Hopkinson, from fig. 5. 6·82 }
$\Delta = 4·12$	8·52	
$\Delta = 3·30$	6·98	
Paraffin wax	2·326	2·32, Boltzmann; 2·32, Northrup; 2·29, Hopkinson; 1·94, v. Pirani.
Bees-wax.....	4·75	3·67 Cavendish.
Shellac	2·49	3·04, Winkelmann; 2·95 to 2·73, Wüllner.
Sealing-wax	{ 4·56 5·2 }	4·3 (Lupton).
Gutta-percha	4·43	2·5 to 4·2 (Lupton).
Chatterton compound	3·98	
Ebonite	2·79	2·76, Schiller; 2·72, Winkelmann; 2·86, Elsas; 2·55, Ferry.
Amber.....	2·80	
Ivory	6·90	
Canada balsam	2·72	
Resin	3·09	3·0, Lombardi.
Soft indiarubber.....	3·08	2·34, Schiller (? hard raw rubber).
Sulphur	4·03	4·05, Fellingner; 4·00, Curie; 3·90, Boltz- mann; 4·2, v. Pirani. Mean of these, 4·03.
Olive oil	3·16	3·16, Hopkinson. Mean of many determina- tions, 3·106 (in Winkelmann).
Heavy paraffin oil, $\Delta = 0·885$	2·55	
Water	∞	75 to 80.